The problem of defining essence

—— including an introduction of Zalta's abstract object theory

This presentation undertakes to investigate the problem of defining essence. The prevailing definition was challenged by Kit Fine's insightful counterexample. Zalta in his [2006] took an attempt to redefine this notion, based on his abstract object theory to solve Fine's counterexample. We may first introduce Zalta's abstract object theory, then articulate Fine's challenge, and finally see Zalta's solution.

The basic thoughts of Zalta's abstract object theory

1. The distinction between exemplify and encode

The ordinary usage of 'exemplify' is predominant in indicating the relationship between objects and properties: If the object *a* has the property *F*, then we say, more formally expressed, that *a* exemplifies *F*. However, most philosophers don't satisfy with this mere kind of notion. For firstly, the problem of whether two properties exemplified exactly the same object are identical is obscure. The notable example by Quine is that, if it is a coincidence that 'the creature with heart' and 'the creature with kidney ' are each exemplified and only exemplified by human, then does it follow that the above properties identical? Problem become more intricate if the discussions concerns abstract objects, which are sharply distinguished with ordinary objects. Of the former, we may raise the question whether an abstract object exist prior to the properties it exemplifies, or the contrary. Or, put it another way, does the object determines its properties or some properties determines an abstract object? Another problem concerns that how to deal with nonexistence and the figments. The most plausible way is to treat them as abstract objects. However, how can we say that an object exemplifies such and such properties, and at the same time asserting that they do not exist? And, while we may be content with the sentence, 'Sherlock Holmes is a figure in Conan Doyle's novels', we may hesitate in truth of the sentence, 'Holmes lived in London ', which connects a fictional object with an actual property.

Noticing that exemplification is just a way of predication, originated in the form of '*a* is *F*', Zalta introduced another predication that would more suit the relationship between abstract objects and properties. Given any property or a set of properties, we would say these properties determine or encode an abstract object. We shall briefly mention the differences between these two kinds of predication. First of all, every property or every combination of properties would encode an unique abstract object. For example, 'gold' and 'being mountain' together determine an abstract object gold mountain; even 'being round' and 'being square' would determine an abstract object the round square. An axiom of O, namely the A-objects axiom, will give a precise picture of this. Secondly, we would probably accept that exemplification is complete, i.e., an object a must exemplify F or its negation. However, this is not the case of encoding. Gold mountain would encode neither 'being higher than the Mount Everest' nor 'not being higher than the Mount Everest'. Thirdly, exemplification is the only predication available for ordinary objects, while abstract objects can be predicated both by exemplification and encoding. Gold mountain would encode 'being gold' and 'being mountain', and at the same time exemplifies the properties such as 'not being located in anywhere of the world' and 'not being a mountain'.

2. The distinction between existence and being concrete

Intuitively, if we suspend the debate over the existence of abstract objects, both abstract and ordinary objects exist, while only ordinary ones are said to be concrete. As we will see later, the theory O has only one fixed domain of objects, regardless of the varying possible worlds. This differentiates it from the Kripke-model, which assigns each possible worlds a different domain of objects. $\forall x\Box\exists y(y=x)$ is a theorem of O, so every objects exists necessarily in every possible worlds. However, the same object would behave itself differently in different possible worlds, due to the properties it exemplifies is not the same. Only objects that exemplify 'being concrete' in some possible worlds would say to be ordinary. 'Being concrete' is characterized as a primitive 1-place relation, denoted as '$E!$', to designate concrete objects within a given possible world. Validating the Barean formula and its converse, $\forall x\Box\varphi\leftrightarrow\Box\forall x\varphi$, we can see more clearly that the world depicted by O are consisted of bare objects, ridded of any properties. It is the properties they exemplify at each possible world that determines their behavior. We may conclude, thereof, existence is not a predicate, which extension would vary from world to world, and we let 'concrete' to take its role in O. This view is compatible with Kant's claim that to exist is not a first-order predicate, and Quine's that to be is to be within the range of the variables.

3. The construction of O

The language would be the second order modal language, plus descriptions and lambda-expressions to represent complex objects and relations. Atomic formulas and formulas that constructed out of them (called 'propositional formulas') are just the same as that of the classical ones, plus another kind of atomic formula of encoding: xF, in order to express this kind of predication. The logical axioms are just the combination of the axioms of propositional calculus, second order logic and S5, except some restrictions due to unaccepted results in the presence of descriptions. Then the proper axioms of O are asserted mainly to rule over ordinary and abstract objects. The model of O has only one fixed domain of objects, and there is no accessibility between possible worlds. In addition, the two kinds of predication are treated extensionally. More specifically, for every n-place relation, there is a exemplification extension which is a ordered n-tuple on the domain of objects; for 1-place relations, there is an additionally encoding extension. Afterwards, the identical objects and relations will be defined.

The Theory of Abstract Objects

A. THE ALPHABET

(1) Primitive object terms
    Names: a_1, a_2,…
    Variables: x_1, x_2,…
(2) Primitive n-place relation terms:
    Names: P^n_1, P^n_2,…, $=_E$, $E!$ $\qquad$ $n\geqslant 0$
    Variables: F^n_1, F^n_2,…
(3) Connectives: $\neg$, $\rightarrow$…
(4) Quantifiers: $\forall$
(5) Lambda: λ
(6) Iota: ι
(7) Box: $\Box$
(8) Parentheses and brackets: (,), [,]

B. FORMULAS AND TERMS

(1) All primitive object terms are object terms; all primitive n-place relation terms are n-place relation terms.

(2) If ρ^0 is any zero-place relation term, ρ^0 is a (propositional) formula.

(3) Atomic exemplification: If ρ^n is any n-place relation term, $o_1, o_2, \ldots, o_n$ are object terms, then $\rho^n o_1 o_2, \ldots o_n$ are (propositional) formula.

(4) Atomic encoding: If ρ^1 is any 1-place relation term, o is object term, then $o\rho^1$ is a formula.

(5) If φ and ψ are formulas, α is any object terms, then $\neg\varphi$, $\varphi\rightarrow\psi$, $\forall\alpha\varphi$, $\Box\varphi$ are (propositional) formulas.

(6) Object descriptions: If φ is any formula with one free variable x, then $(\iota x)\varphi$ is a object term.

(7) Complex n-place relation terms: If φ is any propositional formula, $v_1, v_2, \ldots, v_n$ are any object variables which may or may not occur free in φ, then $[\lambda v_1 v_2 \ldots v_n\ \varphi]$ is a n-place relation term.

C. SEMANTICS

(1) Interpretation:

$\mathfrak{S}$=<$\mathcal{W}$, w_0, $\mathcal{D}$, $\mathcal{R}$, $exl_{\mathcal{W}}$, $\mathcal{L}$, $exl_{\mathcal{A}}$, $\mathcal{F}$>, where:

$\mathcal{W}$ is the domain of POSSIBLE WORLDS.; $w_0 \in \mathcal{W}$ is called the ACTUAL WORLD;

$\mathcal{D}$ and $\mathcal{R}$ are the DOMAIN OF OBJECTS AND RELATIONS, respectively, where $\mathcal{R}=\bigcup_{n\geqslant 0}\mathcal{R}_n$;

$exl_{\mathcal{W}}:\mathcal{R}_n\times\mathcal{W}\rightarrow P(\mathcal{D}^n)$ $(n\geqslant 1)$; $exl_{\mathcal{W}}:\mathcal{R}_0\times\mathcal{W}\rightarrow\{T, F\}$ $(n=0)$; Then exl_w (r^n) is called the EXEMPLIFICATION EXTENSION of r^n at w.

$\mathcal{L}$ contains eight logical functions, $\mathcal{PLUG}_i$, $\mathcal{UNIV}_i$, $\mathcal{CONV}_{i,j}$, $\mathcal{REFL}_{i,j}$, $\mathcal{VAC}_i$, $\mathcal{COND}$, $\mathcal{NEG}$ and $\mathcal{NEC}$, all of which are on the domain of relations and objects to form new relations, a correspondence of λ-expressions:

(a) for each $i\geqslant 1$, $\mathcal{PLUG}_i:\mathcal{R}_n\times\mathcal{D}\rightarrow\mathcal{R}_{n-1}$
for $n>1$, $exl_w(\mathcal{PLUG}_i(r^n,o))=\{<o_1,\ldots,o_{i-1},o_{i+1},\ldots,o_n> : <o_1,\ldots,o_{i-1},o,o_{i+1},\ldots,o_n>\in exl_w(r^n)\}$
for $n=1$, $exl_w(\mathcal{PLUG}_i(r^1,o))=\begin{cases}\text{T iff } o\in exlw\ (r1)\\ F,\ \text{otherwise}\end{cases}$

(b) for each $i\geqslant 1$, $\mathcal{UNIV}_i:\mathcal{R}_n\rightarrow\mathcal{R}_{n-1}$
for $n>1$, $exl_w(\mathcal{UNIV}_i(r^n))=\{<o_1,\ldots,o_{i-1},o_{i+1},\ldots,o_n> : \forall o(<o_1,\ldots,o_{i-1},o,o_{i+1},\ldots,o_n>\in exl_w(r^n))\}$
for $n=1$, $exl_w(\mathcal{UNIV}_i(r^1))=\begin{cases}\text{T iff } \forall o(o\in exlw\ (r1))\\ F,\ \text{otherwise}\end{cases}$

(c) for each i,j, $1\leqslant i<j$, $\mathcal{CONV}_{i,j}:\mathcal{R}_n\rightarrow\mathcal{R}_n$
$exl_w(\mathcal{CONV}_{i,j}(r^n))=\{<o_1,\ldots,o_{i-1},o_j,o_{i+1},\ldots, o_{j-1},o_i,o_{j+1},\ldots,o_n> : <o_1,\ldots,o_i,\ldots,o_j,\ldots,o_n>\in exl_w(r^n)\}$

(d) for each i,j, $1\leqslant i<j$, $\mathcal{REFL}_{i,j}:\mathcal{R}_n\rightarrow\mathcal{R}_{n-1}$
$exl_w(\mathcal{REFL}_{i,j}(r^n))=\{<o_1,\ldots,o_i,\ldots, o_{j-1},o_{j+1},\ldots,o_n> : <o_1,\ldots,o_i,\ldots,o_j,\ldots,o_n>\in exl_w(r^n)$ and $o_i=o_j\}$

(e) for each $i\geqslant 1$, $\mathcal{VAC}_i:\mathcal{R}_n\rightarrow\mathcal{R}_{n+1}$
for $n\geqslant 1$, $exl_w(\mathcal{VAC}_i(r^n))=\{<o_1,\ldots,o_{i-1},o.o_i,o_{i+1},\ldots,o_n> : <o_1,\ldots,o_i,\ldots,o_n>\in exl_w(r^n)\}$
for $n=0$, $exl_w(\mathcal{VAC}_i(r^0)=\{o : exl_w(r^0)=T\}$

(f) $\mathcal{COND}:\mathcal{R}_n\times\mathcal{R}_m\rightarrow\mathcal{R}_{n+m}$
for $n\geqslant 1$, $m\geqslant 1$, $exl_w(\mathcal{COND}(r^n, s^m))=\{<o_1,\ldots,o_n,o_1',\ldots,o_m'> : <o_1,\ldots,o_n,>\notin exl_w(r^n)$ or $<o_1',\ldots,o_m'>\in exl_w(s^m)\}$
for $n=0$, $m\geqslant 1$, $exl_w(\mathcal{COND}(r^0, s^m))=\{<o_1,\ldots,o_m> : exl_w(r^0)=F$ or

$<o_1,\ldots,o_m> \in exl_w$ (s^m) }

for n⩾1, m=0, exl_w ($\mathcal{COND}(r^n, s^0)$)={$<o_1,\ldots,o_n>$: $<o_1,\ldots,o_n> \notin exl_w$ (r^n) or exl_w (s^0) =T}

for n=0, m=0, exl_w ($\mathcal{COND}(r^0, s^0)$)=$\begin{cases} \text{T iff } exlw\ (\text{rn}) = F \text{ or } exlw\ (\text{sm}) = \text{T} \\ F, \text{otherwise} \end{cases}$

(g) $\mathcal{NEG}:\mathcal{R}_n \to \mathcal{R}_n$

for n⩾1, exl_w ($\mathcal{NEG}(r^n)$)={$<o_1,\ldots,o_n>$: $<o_1,\ldots,o_n> \notin exl_w$ (r^n)}

for n=0, exl_w ($\mathcal{NEG}(r^0)$)=$\begin{cases} \text{T iff } exlw\ (\text{r0}) = F \\ F, \text{otherwise} \end{cases}$

(h) $\mathcal{NEC}:\mathcal{R}_n \to \mathcal{R}_n$

for n⩾1, exl_w ($\mathcal{NEC}(r^n)$)={$<o_1,\ldots,o_n>$: $\forall w'(<o_1,\ldots,o_n> \in exl_{w'}$ (r^n)}

for n=0, exl_w ($\mathcal{NEC}(r^0)$)=$\begin{cases} \text{T iff } \forall w'(exlw'\ (\text{r0}) = \text{T}) \\ F, \text{otherwise} \end{cases}$

$exl_{\mathcal{A}}:\mathcal{R}_1 \to P(\mathcal{D})$; Then $exl_{\mathcal{A}}$ (r^1) is called the ENCODING EXTENSION of r^1.

$\mathcal{F}(\kappa) \in \mathcal{D}$ for each object name κ; $\mathcal{F}(\kappa_n) \in \mathcal{R}_n$ for each relation name κ_n.

(2) ASSIGNMENTS

Where v is any object variable, $f(v) \in \mathcal{D}$;

Where r^n is any relation terms: $f(r^n) \in \mathcal{R}_n$;

(3) DENOTATIONS

First of all, we shall note that given an arbitrary λ-expression, we may have more than one way to interpret it[1]. To get rid of this defect, we introduce the method of partitioning λ-expressions into eight equivalent classes, as follows:

Let $\mu=[\lambda v_1 \ldots v_n\ \varphi]$ be an arbitrary λ-expression

(a) If there is 1⩽i⩽n such that v_i does not occur free in φ and v_i is the least such variable, then μ is the *i-th vacuous expansion* of $[\lambda v_1 \ldots v_{i-1} v_{i+1} \ldots v_n\ \varphi]$

(b) If it is not the case of (a), and if there is 1⩽i⩽n such that v_i is not the i-th free object variable in φ and v_i is the least such variable, then μ is the *i,j-th conversion* of $[\lambda v_1 \ldots v_{i-1} v_j v_{i+1} \ldots v_{j-1} v_i v_{j+1} \ldots v_n\ \varphi]$, where v_j is the i-th free object variable in φ

(c) If μ is neither of the above, then if there is 1⩽i⩽n such that v_i occurs more than once in φ and v_i is the least such variable, then μ is the *i,j-th reflection* of $[\lambda v_1 \ldots v_{i+k} v v_j \ldots v_n\ \varphi']$, where k is the number of object variables between the first and second occurrences of v_i, φ' is the result of replacing the second occurrence of v_i with a new variable v, and j=i+k+1.

(d) If μ is none of the above, then

(i) If $\varphi=\neg\psi$, then μ is the *negation* of $[\lambda v_1 \ldots v_n\ \psi]$;

(ii) If $\varphi=\psi\to\gamma$, $v_1,\ldots,v_p$ and is the free object variables of ψ, $v_{p+1},\ldots,v_n$ is the free object variables of γ, then μ is the *conditionalization* of $[\lambda v_1 \ldots v_p\ \psi]$ and $[\lambda v_{p+1} \ldots v_n\ \gamma]$;

(iii) If $\varphi=\forall v\psi$, v is the i-th free object variable, then μ is the *i-th universalization* of $[\lambda v_1 \ldots v_{i-1} v v_i v_{i+1} \ldots v_n\ \psi]$

(iv) If $\varphi=\Box\psi$, then μ is the *necessitation* of $[\lambda v_1 \ldots v_n\ \psi]$

(e) If μ is none of the above, o is the left most object term occurring in φ, then μ is the *i -th plugging* of $[\lambda v_1 \ldots v_j v v_{j+1} \ldots v_n\ \varphi']$, where j is the number of object variables occurring before o, φ' is the result of replacing the first occurrence of o with a new variable v, and i=j+1

(f) If μ is none of the above, then φ is atomic, $v_1,\ldots,v_p$ is in order in which these variables first occur in φ, $\mu=[\lambda v_1 \ldots v_n\ \rho^n v_1 \ldots v_n]$ for some relation term ρ^n,

[1] For example, given an interpretation $\mathfrak{S}$ and assignment f, $[\lambda x\ Fx \to Gyx]$ can be interpreted in the following three way (supposing that $f(F)$=R, $f(G)$=S, $f(y)$=o): (i) $\mathcal{REFL}_{1,2}(\mathcal{COND}(\text{R}, \mathcal{PLUG}_1(\text{S, o})))$; (ii) $\mathcal{PLUG}_2(\mathcal{REFL}_{1,3}(\mathcal{COND}(\text{R, S})), \text{o})$; (iii) $\mathcal{REFL}_{1,2}(\mathcal{PLUG}_2(\mathcal{COND}(\text{R, S}), \text{o}))$

and μ is called *elementary*.

Given the interpretation $\mathfrak{S}$ and assignment f, then the denotation d of term τ with respect to $\mathfrak{S}$ and f is:

(a) $d(\kappa)=\mathcal{F}(\kappa)$, where κ is a primitive name;
(b) $d(\alpha)=f(\alpha)$, where α is a primitive variable;
(c) where $(\iota x)\varphi$ is a object description,

$$d\big((\iota x)\varphi\big)^{2}=\begin{cases}\text{o iff } (\exists f')(f' x f \wedge f'(x)=\text{o} \wedge f' \text{ satisfies } \varphi \text{ with respect to} \\ w0 \wedge (\forall f'')(f'' x f' \wedge f'' \text{ satisfies } \varphi \text{ with respect to } w0 \rightarrow f''=f')) \\ \text{undefined, otherwise}\end{cases}$$

(d) where $[\lambda v_1 v_2 \ldots v_n \;\; \rho^n v_1 v_2 \ldots v_n]$ is the elementary λ-expression, $d([\lambda v_1 v_2 \ldots v_n \;\; \rho^n v_1 v_2 \ldots v_n])=d(\rho^n)$
(e) where μ is the i-th plugging of ξ by o, $d(\mu)=\mathcal{PLUG}_i(d(\xi),d(o))$
(f) where μ is the i-th universaliation of ξ, $d(\mu)=\mathcal{UNIV}_i(d(\xi))$
(g) where μ is the i,j-th conversion of ξ, $d(\mu)=\mathcal{CONV}_{i,j}(d(\xi))$
(h) where μ is the i,j-th reflection of ξ, $d(\mu)=\mathcal{REFL}_{i,j}(d(\xi))$
(i) where μ is the i-th vacuous expansion of ξ, $d(\mu)=\mathcal{VAC}(d(\xi))$
(j) where μ is the conditionalization of ξ by ζ, $d(\mu)=\mathcal{COND}(d(\xi),d(\zeta))$
(k) where μ is the negation of ξ, $d(\mu)=\mathcal{NEG}(d(\xi))$
(l) where μ is the necessitation of ξ, $d(\mu)=\mathcal{NEC}(d(\xi))$

(4) SATISFACTION

(a) If φ is any primitive zero-place term, f satisfies φ with respect to w iff $exl_w(d(\varphi))$=T;

(b) If $\varphi=\rho^n o_1 o_2,\ldots o_n$, f satisfies φ with respect to w iff $(\exists \text{o}_1)\ldots(\exists \text{o}_n)(\exists \text{r}^n)(\text{o}_1=d(o_1)\wedge\ldots\wedge=d(o_2)\wedge \text{r}^n=d(\rho^n)\wedge<\text{o}_1,\ldots,\text{o}_n>\in exl_w(d(\text{r}^n))$

(c) If $\varphi=o\rho$, f satisfies φ with respect to w iff $(\exists \text{o})(\exists \text{r}^1)(\text{o}=d(o)\wedge \text{r}^1=d(\rho)\wedge \text{o}\in exl_A(d(\text{r}^1))$

(d) If $\varphi=\neg\psi$, or $\varphi=\psi\rightarrow\gamma$, or $\varphi=\forall\alpha\psi$, f satisfies φ with respect to w according to usual definition.

(e) If $\varphi=\Box\psi$, f satisfies φ with respect to w iff $\forall w'$(f satisfies ψ with respect to w')

D. THE LOGIC

(1) LOGICAL AXIOMS:

Definition 1: x is ABSTRACT (A!x): $[\lambda y\ \Box\neg E!y]x$
Definition 2: x is ORDINARY (O!x): $\Diamond E!x$
Definition 3: $F^1=G^1$: $\forall x(xF\leftrightarrow xG)$

Prepositional Schemata:

LA1: $\varphi\rightarrow(\psi\rightarrow\varphi)$
LA2: $(\varphi\rightarrow(\psi\rightarrow\gamma))\rightarrow(\varphi\rightarrow\psi)\rightarrow(\varphi\rightarrow\gamma)$
LA3: $(\neg\varphi\rightarrow\neg\psi)\rightarrow((\neg\varphi\rightarrow\psi)\rightarrow\varphi)$

Quantificational Schemata:

LA4[3]: (a) $\forall\alpha\varphi\rightarrow\varphi^{\tau}_{\tau}$, where τ contains no descriptions and is substitutable for α
(b) $\forall\alpha\varphi\rightarrow(\psi^{\tau}_{\beta}\rightarrow\varphi^{\tau}_{\alpha})$, where ψ is atomic, and τ both contains a description and is substitutable for α,β

[2] Ii is a very important feature that we interpret our descriptions rigidly, that is , we pick out x through ψ only according to the actual world w_0.

[3] The original axiom:'$\forall\alpha\varphi\rightarrow\varphi^{\tau}_{\tau}$, where τ is substitutable for α' is thus restricted to avoid undenoting descriptions. Consider the following inference: (i) $\forall x(x=x)$ by the theorem of quantificational logic; (ii) $(\iota x)Gx=(\iota x)Gx$ by (i) and the original LA4; (iii) $\exists y(y=(\iota x)Gx)$ by (ii) and Existential Introduction. (iii) is clearly not valid. If *G* is the property of being the round square, nothing exists would exemplify this property. However, if such terms occur true in an atomic formula, we would have no such worries. Hence ,LA4b is conditioned by an atomic formula ψ.

LA5: $\forall\alpha(\varphi\rightarrow\psi)\rightarrow(\varphi\rightarrow\forall\alpha\psi)$

Modal Schemata

LA6: $\Box\varphi\rightarrow\varphi$ (T)

LA7: $\Box(\varphi\rightarrow\psi)\rightarrow(\Box\varphi\rightarrow\Box\psi)$ (K)

LA8: $\Diamond\varphi\rightarrow\Box\Diamond\varphi$ (5)

LA9: $\Box\forall x\varphi\leftrightarrow\forall x\Box\varphi$ (Barean formula)

LA10: $\forall x\forall F(\Diamond xF\rightarrow\Box xF)$

The last axiom of this schemata is important for abstract objects. It asserts that an object encodes the same properties in every possible worlds, and that it encodes necessarily.

λ-Schemata

λ-EQUIVALENCE (also λ-conversion)

where φ is any propositional formula with no description,

$\forall x_{1\ldots}\forall x_{\mathrm{n}}([\lambda v_{1\ldots}v_{\mathrm{n}}\varphi]x_1\ldots x_{\mathrm{n}}\leftrightarrow\varphi^{x1,\ldots,x\mathrm{n}}_{\mathrm{v1,\ldots vn}})$

λ-IDENTITY

$[\lambda v_{1\ldots}v_{\mathrm{n}}\ \rho^{\mathrm{n}}v_{1\ldots}v_{\mathrm{n}}]=\rho^{\mathrm{n}}$ and $[\lambda v_{1\ldots}v_{\mathrm{n}}\ \rho^0]=[\lambda v'_1\ldots v'_{\mathrm{n}}\rho^0]$, where $v_{1\ldots}v_{\mathrm{n}}$ and $v'_1\ldots v'_{\mathrm{n}}$ are distinct object variables.

Description Schemata[4]:

L-DESCRIPTIONS1:

where ψ is atomic, $\psi^{(\iota x)\varphi}_v\rightarrow\exists y(\varphi^y_x\wedge\psi^y_v)$

L-DESCRIPTIONS2:

where ψ is atomic, $\psi^{(\iota x)\varphi}_v\rightarrow\forall y(\varphi^y_x\rightarrow\psi^y_v)$

L-DESCRIPTIONS3:

where ψ is atomic with v_1 free and γ with v_2 free,

$\psi^{(\iota x)\varphi}_{v1}\rightarrow\neg(\exists y(\varphi^y_x\wedge\gamma^y_{v2})\wedge\exists y(\varphi^y_x\wedge\neg\gamma^y_{v2}))$

(2) RULES OF INFERENCE

(a) MP: (b) Universal Introduction ('UI'): $\vdash\varphi\Rightarrow\vdash\forall\alpha\varphi$

(c) Necessitation Introduction (derived and restricted[5]) ('$\Box$I'): If we are given a proof of φ from a set of formulas Γ, then if the proof φ does not depend on any unmodalized formulas of Γ, then $\Gamma\vdash\Box\varphi$.

(d) Second-order comprehension schema for relations ('RELATION'): $\exists F^{\mathrm{n}}\Box\forall x_{1\ldots}\forall x_{\mathrm{n}}(F^{\mathrm{n}}x_{1\ldots}x_{\mathrm{n}}\leftrightarrow\varphi)$, where φ is propositional and contains no free F^{n} 's and descriptions.

(3) PROPER AXIOMS

A1. ('E-IDENTITY'): $x{=}_E y\leftrightarrow(\Diamond E!x\wedge\Diamond E!y\Box\forall F(Fx\leftrightarrow Fy))$

A2. ('NO-CODER'): $\Diamond E!x\rightarrow\Box\neg(\exists FxF)$

Hence ordinary objects do not encode any property.

Definition 4 (for identity): $x{=}y$: $x{=}_E y\vee(\mathrm{A}!x\wedge\mathrm{A}!y\wedge\Box\forall F(xF\leftrightarrow yF))$

After this uniqueness can be defined. $\exists!x\varphi(x)$:

$\exists x(\varphi(x)\wedge\forall y(\varphi(y)\rightarrow(x{=}_E y\vee(\mathrm{A}!x\wedge\mathrm{A}!y\wedge\Box\forall F(xF\leftrightarrow yF)))))$

A3. ('IDENTITY'): $\alpha{=}\beta\rightarrow\varphi(\alpha,\alpha)\leftrightarrow\varphi(\alpha,\beta)$, where $\varphi(\alpha,\beta)$ is the result of replacing some, but not necessarily all, free occurrences of α by β in $\varphi(\alpha,\alpha)$,, provided it is substitutable.

A4. ('A-OBJECTS'): $\exists x(\mathrm{A}!x\wedge\forall F(xF\leftrightarrow\varphi))$, where x is not free in φ.

It is an important axiom for abstract objects, which asserts that any condition expressed by φ determines an unique abstract objects[6].

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[4] Similarly, the axioms of this schemata are all conditioned by an atomic formula to avoid undenoting descriptions.

[5] Remember that our interpretation of description is rigid (see footnote 2), this restriction is to prevent the following derivation which is not true in all interpretations: (i) $F(\iota x)Gx\rightarrow\exists y(Gy)$ by L-DESCRIPTIONS1; (ii) $\Box(F(\iota x)Gx\rightarrow\exists y(Gy))$ by (i) and the unrestricted version of $\Box$I; (iii) $\Box F(\iota x)Gx\rightarrow\Box(\exists y(Gy))$ by LA7 and (ii). (iii) asserts that if there is a unique object exemplifies G in w_0 that exemplifies F necessarily, then there is an object exemplifies G necessarily. Clearly, it is not true in any interpretation.

[6] Suppose both x and y are abstract objects determined by φ, then x and y encode exactly the same properties;

A5. ('DESCRIPTIONS'): $\psi_v^{(\iota x)\varphi} \rightarrow (\exists! y \varphi_x^y \wedge \exists y(\varphi_x^y \wedge \psi_v^y))$, where ψ is atomic with one free object variable v

Defining Essence

1. The problem

Essential properties has always been the center of metaphysics. Early philosophers conceived essential properties as that of being exemplified by an object whenever the object exists. After modal logic and the possible worlds semantic has developed, the notion can be represented in a more formalized manner. An property F is essential to an object x, is defined as x exemplifies F in every possible worlds whenever x exists, formalized as, $\Box(\exists y(y{=}x) \rightarrow Fx)$.. The definition has its virtue in elegance and intuitive However, the two counterexamples devised by Kit Fine in his [1994a] seems to imply that this definition of essence (abbreviated as '(E)') is too simplistic.

Take a look of his first counterexample. Consider two objects, Socrates ('s') and singleton Socrates ('{s}'), one may prove, using (E) and the modal set theory, and assuming that the property of having Socrates as a member is essential to singleton Socrates, that the property of being a member of singleton Socrates is essential to Socrates, a consequences that we wish to avoid. More specifically, suppose $[\lambda y\ \mathrm{s} \in y]$ is essential to {s}, then by (E), $\Box(\exists y(y{=}\{\mathrm{s}\}) \rightarrow [\lambda y\ \mathrm{s} \in y]\{\mathrm{s}\})$. By λ-conversion, $\Box(\exists y(y{=}\{\mathrm{s}\}) \rightarrow \mathrm{s} \in \{\mathrm{s}\})$. And by modal set theory, we may have $\Box(\exists y(y{=}\{\mathrm{s}\}) \leftrightarrow \exists y(y{=}\mathrm{s}))$, hence $\Box(\exists y(y{=}\mathrm{s}) \rightarrow \mathrm{s} \in \{\mathrm{s}\})$. Again, by λ-conversion, $\Box(\exists y(y{=}\mathrm{s}) \rightarrow [\lambda y\ y \in \{\mathrm{s}\}]\mathrm{s})$, finally we conclude that $[\lambda y\ y \in \{\mathrm{s}\}]$ is essential to s by (E).

The second counterexample is stronger in the sense that it give rise to similar results even without applying to modal set theory.. Consider two *unconnected* objects, say, Socrates ('s') and the Eiffel Tower ('t'). For one thing, if being distinct from Eiffel Tower is essential to Socrates, then $\Box(\exists y(y{=}\mathrm{s}) \rightarrow [\lambda y\ y \neq \mathrm{t}]\mathrm{s})$, then there is something that connects Socrates and Eiffel Tower, namely the necessary distinction. Hence being distinct from Eiffel Tower is not essential to Socrates. However, on the other hand, if we can show that $x \neq y \rightarrow \Box x \neq y$ is a theorem of S5, then by the assumption that $\mathrm{s} \neq \mathrm{t}$, we get $\Box \mathrm{s} \neq \mathrm{t}$ and $\Box[\lambda y\ y \neq \mathrm{t}]\mathrm{s}$. Finally, $\Box(\exists y(y{=}\mathrm{s}) \rightarrow [\lambda y\ y \neq \mathrm{t}]\mathrm{s})$, which is contrary to the intuition above. The following is a proof of $x \neq y \rightarrow \Box x \neq y$:

1. $x{=}y$	Assumption	
2. $\Box x{=}y$	$\Box$I	
3. $x{=}y \rightarrow \Box x{=}y$	1,2, DE	
4. $\Box(x{=}y \rightarrow \Box x{=}y)$	3, $\Box$I	
5. $\Box(x{=}y \rightarrow \Box x{=}y) \rightarrow (\Diamond x{=}y \rightarrow \Diamond\Box x{=}y)$		Theorem
6. $\Diamond x{=}y \rightarrow \Diamond\Box x{=}y$	MP	
7. $\Diamond x{=}y$	Assumption	
8. $\Diamond\Box x{=}y$	6, MP	
9. $\Diamond x{=}y \rightarrow \Diamond\Box x{=}y$	7,8 DE	
10. $\Diamond\Box x{=}y \rightarrow \Box x{=}y$	5	
11. $\Box x{=}y \rightarrow x{=}y$	T	
12. $\Diamond x{=}y \rightarrow x{=}y$	9, 10, 11	
13. $x \neq y \rightarrow \neg\Diamond x{=}y$	12, Contraposition	
14. $x \neq y \rightarrow \Box x \neq y$		

Why cannot we accept that being a member of singleton Socrates is essential to Socrates? For one thing, epistemologically speaking, we mainly learn, perceive and

Then by LA10, they encode necessarily. Using the definition for identity, we can conclude that $x{=}y$.

differentiate objects through their properties, among which the essential properties play an special role. Intuitively, essence is the form of the objects that cannot be deprived of. We might probably say that only when one acquire the essence of something, can he said to be fully master or understand the object. One may in his early age knows what a circle looks like and what is similar to circles, but he might not said to be understand what a circle is until he learned the definition of circles in geometry. Following this line, if being the member of singleton Socrates is really essential to Socrates, then one has to know that Socrates is a member of singleton Socrates to fully understand Socrates.. But singleton Socrates, under which lies a whole and complicated conventions and techniques of set theory, occurred a long time after Socrates' birth and death. The consequence is that nobody can truly know who Socrates is until set theory is developed, of which we cannot accept. This 'Socrates-counterexample' can be extended to reveal further difficulties, as suggested by Fine, by replacing s$\in$\{s\} by any necessary mathematical truth. Then by the same token, it is necessary that this truth holds whenever Socrates exists, and this truth may become part of Socrates' essence. But it is preposterous to conclude that one has to know all the truths in mathematics before he can fully understand Socrates.

Fine's second counterexample, though not assuming any modal set theory, has nevertheless invoked an ambiguous term 'unconnected'. We may make up the definition in a coarse manner: s and t are unconnected if and only if $\neg\exists F(F\text{st}\vee F\text{ts})$; or $\exists y(y=\text{s})\wedge\neg\Diamond(\exists y(y=\text{s})\rightarrow\neg\exists y(y=\text{t}))$.

Fine thought of these difficulties of the notion of essence as it connects to modal notions and devised a logic of essence to solve the problem (Fine, [1995]). Zalta's solution took another path. First of all, he assimilated the notion of essence into his logic of abstract object rather than devising a separate logic as Fine did. Secondly, he made distinct definitions for ordinary objects and abstract objects respectively. For the ordinary objects, he maintained the modal notions and replaced the 'existence' in (E) by a more fine-grained notion 'concrete'; for abstract objects, he used the notion of encoding. This practice are claimed to be able to contain the above two counterexamples. Let's see how.

2. Defining the essence of ordinary objects

This definition is very similar to (E), except that it replaces $\exists y(y=x)$ by a primitive relation symbol '$E!$'of O as follows:

Essential$(F,x)=_{df}\Box(E!x\rightarrow Fx)$.

So, 'F is essential to x' is defined as necessarily x amplifies F whenever x is concrete, rather than necessarily x exemplifies F whenever x exists. The distinction between 'existence' and 'concrete' is briefly mentioned before. O treats every objects as necessary existence, though every objects behave differently in different possible worlds due to their relationships with properties they exemplify or encode. 'Concrete' can be coarsely understood as 'being spatiotemporal' or 'having mass and extension and being continuous in time'. The above definition can turn out to be more subtle if distinguished from the properties that x exemplifies at every possible worlds, e.g., the property of being self-identical. If F is the property of being self-identical, then obviously, $\Box Fx$ regardless of x concrete or not. So we may define 'F is strong essential to x' as follows:

SEssential$(F,x)=_{df}$ Essential$(F,x)\wedge\neg\Box Fx$

Since Fine's second counterexample concerns ordinary objects only, we may first take a look at it. Assume that s and t are unconnected, then s is distinct from t, formalized in O as, s$\neq_E$t, one may show that $\Box$s$\neq_E$t, hence $\Box[\lambda y\ y\neq_E\text{t}]$s; also, $\Box(E!\text{s}\rightarrow[\lambda y\ \ y\neq_E\text{t}]\text{s})$ holds, so Essential($[\lambda y\ \ y\neq_E\text{t}]$,s). Finally, we have $\neg$SEssential($[\lambda y\ y\neq_E\text{t}]$,s). In this way, the intuition that being distinct from the Eiffel Tower is not essential to Socrates remains.

However, this solution is flawed. (1) we have just proved that being distinct from the Eiffel Tower is not Strong Essential to Socrates, but the weak essence remains. Even if we are content with this result, we can by the very same manner define (SE) on the basis of (E):

(SE) F is strong essential to $x =_{df} \Box(\exists y(y=x) \to Fx) \wedge \neg\Box Fx$

And the conclusion would be very much the same. Then why should we adopt O to redefine the notion? (2) The definition depends on the ambiguous term 'concrete'. Remember that 'E!' is primitive in O, and that nothing in the semantics that give E! a special role, and the axioms that includes E! have not give us any sense exactly how does E! behaves. So it seems that the solution does not eradicate the problem, it just transfers the difficulties from defining 'essence' to defining 'concrete'. (3) There is a more important defects. Let H be the conjunction of two properties F and G, where SEssential(F,x) and $\Box Gx$, then by the definition above, one may easily show that SEssential(H,x). Instantiating the result, the conjunctive property of being human (which we may assume to be strongly essential to Socrates) and not being identical to the Eiffel Tower is also strongly essential to Socrates. The unintuitive result then recovers.

3. Defining the essence of abstract objects

The axiom A-objects

$\exists x(\text{A}!x \wedge \forall F(xF \leftrightarrow \varphi))$, where x is not free in φ

tells us the existence of an abstract object corresponding to any condition on properties expressible in the language. Then by the definition for identity of objects:

$x=y$: $x =_E y \vee (\text{A}!x \wedge \text{A}!y \wedge \Box\forall F(xF \leftrightarrow yF))$

one may easily show that the abstract object thus determined by φ is unique. Again remember the satisfaction for encoding formulas:

If $\varphi = o\rho$, f satisfies φ with respect to w iff

$(\exists \text{o})(\exists \text{r}^1)(\text{o}=d(o) \wedge \text{r}^1=d(\rho) \wedge \text{o} \in exl_A(d(\text{r}^1)))$

where the encoding extension is world-free; and the axiom:

$\forall x \forall F(\Diamond xF \to \Box xF)$

we can see that if an abstract object a encodes F at a possible world w, then a encodes F at every possible worlds. In other words, an abstract object a is uniquely determined by a property or the combination of properties, without varying from world to world.

Thus , intuitively, the properties that an abstract object encode is exactly the ones that are essential to it. For. suppose $F=F_1 \wedge \ldots \wedge F_n$, by A-objects, determines an abstract object a. If there is some F_i that is not the essence for a, then intuitively, there are cases (or possible worlds) in which there is a without having F_i. But it then violates the consequence that a encodes F at every possible worlds. Or, if there is another distinct property G such that aG and G is essential for a, then a thus determined by $F \wedge G$ is identical to a determined by F alone, violating the A-objects. Only one question remains: would some properties exemplified by a be essential? We may introduce the concept of 'concreteness-entailing' to analyze this. A property F is concreteness entailing (CE(F)) if and only if $\Box\forall x(Fx \to E!x)$. That is , one an object exemplifies a concreteness-entailing object F, it follows that it will also amplifies the property of being concrete. 'Being red', 'being heavy', 'being somebody's sister' are all concreteness-entailing properties. Here, we assert that all abstract objects exemplify the same properties, namely all properties other than the concreteness-entailing ones. Among these properties, there are intentional ones, e.g., 'being conceived of', being thought of'[7]. Having noted this, we can conclude that the

[7] Though it is disputable that 'the round square' which is determined by 'being round' and 'being square' can be conceived of, here we assume that every abstract objects can be conceived of, guaranteed by A-objects that asserts its existence.

properties encoded by an abstract object are essential for them. So we thus define the essential properties for abstract objects:

Essential$_A$ $(F,x) =_{df} xF$

Now we can turn to analyze the first counterexample by Fine concerning Socrates and singleton Socrates. However, in O, the term ‘singleton Socrates’ is ambiguous until we supply a context. Another step should be adopted to import the theory of modal set theory (abbreviated as ‘M’) into the present theory O. We use the following two steps for importation: first we index every term of M, e.g., $\emptyset$ of M is indexed as $\emptyset_M$, then we may have the indexed formulas of O, e.g., $\emptyset_M \notin \emptyset_M$; then we treat the theory M as an abstract object that encodes propositional properties of the form being such that p (‘$[\lambda y\ p]$’), where p is any indexed true sentence according to M. Therefore, a sentence p true according to theory M is defined as follows:

$M \vDash p_M =_{df} M[\lambda y\ p_M]$, where p_M is the indexed formula

We then introduce the following two principles:

Theoretical Identification Principle:

$\kappa_M = (\iota x)(A!x \wedge \forall F(xF \leftrightarrow M \vDash F\kappa_M))$, where κ_M is any name occurring in M.

This principle asserts that κ_M is the abstract object that encodes exactly the properties exemplified κ_M by according to M.

Metatheoretic Importation Principle:

If $M \vDash \varphi$, then $O \vdash M \vDash \varphi_M$

From the above definition and principles, we immediately get the following theorem:

Equivalence Theorem:

$\kappa_M F \leftrightarrow M \vDash F\kappa_M$

Now we have the following theorems of O:

$O \vdash M \vDash \emptyset_M \notin \emptyset_M$

$O \vdash M \vDash \emptyset_M \in \{\emptyset_M\}$

$O \vdash \emptyset_M[\lambda y\ y \notin \emptyset_M]$

$O \vdash \emptyset_M[\lambda y\ y \in \{\emptyset_M\}]$

Nevertheless, we shall not forget that what we have imported into O are a theory M together with all the sentences true according to M, hence ‘$\emptyset_M \notin \emptyset_M$’ only is not the theorem of O. Then ‘singleton Socrates’ is indexed as ‘$\{s\}_M$’; Socrates, being a ordinary object, can be just represented as ‘s’ without indexing. By Theoretical Identification Principle, $\{s\}_M = (\iota x)(A!x \wedge \forall F(xF \leftrightarrow M \vDash F\{s\}_M))$.

By M, $M \vDash s \in \{s\}_M$, and by Metatheoretic Principle, $O \vdash M \vDash s \in \{s\}_M$, then by Equivalence Theorem, $O \vdash \{s\}_M[\lambda y\ s \in y]$. Finally, using the definition of essence for abstract objects, we conclude that having Socrates as a member is essential for singleton Socrates.

On the other hand, ‘$s \in \{s\}_M$’ alone is not a theorem of O, so none of the following are theorems of O:

$O \nvdash [\lambda y\ y \in \{s\}_M]s$

$O \nvdash \Box(E!s \rightarrow [\lambda y\ y \in \{s\}_M]s)$

$O \nvdash$ Essential$([\lambda y\ y \in \{s\}_M],s) \vee$ SEssential$([\lambda y\ y \in \{s\}_M],s)$

Thus, the property of being a member of singleton Socrates is neither strong nor weak essential to Socrates.

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